SHOULD I TAKE CS 20?

This quiz is meant to help you decide whether to take CS 20. If the terminology and questions are unfamiliar to you, or if they are vaguely familiar but you can't actually do the problems, you should take CS 20 before taking CS 121 or CS 124 or AM 107. If it's all easy for you, you are ready to go into those courses. If it's half-and-half, you probably should take CS 20.

Even if some topics are unfamiliar to you, you should not take CS 20 if you have taken a course such as Math 23 or 25 or 55. You won't have any trouble filling in the gaps if your mathematical experience is at that level.

One of the skills CS 20 will develop is writing mathematics. That can't be tested easily on a quiz like this, but if you've never written a formal mathematical argument carefully, CS 20 will teach you how to do it, and that is an important skill to have for later CS courses.

1. Let p mean "Anne is sick," q mean "Anne misses the final exam," and r mean "Anne passes the course." Write a Boolean formula (formula of propositional logic) stating that Anne passes the course unless she misses the final exam and isn't sick.

2. Put into disjunctive (or-of-ands) form: $(p \vee q) \wedge (\neg(q \wedge \neg r))$.

3. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology using any formal method you wish.

4. Let $P(x)$ mean "x is a politician," $H(x)$ mean "x is honest," and $V(x)$ mean "x votes for raising the debt ceiling." Write a formula of first-order logic (the quantificational calculus) stating that no honest politician would vote to raise the debt ceiling.

5. Let $P(x, y, z)$ mean that z is the product of x and y, all integers. Write a formula that means that x is a prime number. You may use the $>$, $<$, and $=$ signs, and numerals such as 1, 2, 3.

6. Prove that the product of two odd integers is odd.

7. Prove that you can can't color the sides of a cube with only two colors in such a way that no two adjacent sides are the same color. Do three colors suffice to make such a coloring possible?

8. Prove by induction that the sum of the first k odd numbers is a perfect square, for any k.

9. What is $\sum_{k=1}^{n} 2^{-k}$ when $n = 2$? Prove that the sum is less than 1 for any n.

10. Let O be the set of odd numbers, E the set of even numbers, and P the set of prime numbers. What are the sets $O \cup E$, $E - P$, and $P \cap O$?

11. How many nonempty subsets does a set of size 3 have?

12. How many permutations of the string $abcdef$ are there? How many of those have the substring cde (as a contiguous block of three letters)? How many have the c, d, and e in that order but not necessarily contiguous?

13. Facebook uses 64-bit identifier codes for users, pictures, and other data objects. They never re-use an identifier. How many would they have to use per day to run out within a century?

14. How many different strings can be made by rearranging the letters in the word OBAMA?

15. Write a recurrence relation for $E(n)$, the number of bit strings of length n that have an even number of 0s.

16. True or false?
    (a) $3000n^2 + 22n + 1 = O(n^2)$
    (b) $3000n^2 + 22n + 1 = o(n^2)$
    (c) The number of even-sized subsets of a set of n elements is O(the number of all subsets of the set), and *vice versa.*

17. What does it mean to say that $f(n) = n^{O(1)}$?

18. A roulette wheel has 38 numbers: 18 black, 18 red, plus 0 and 00. What is the probability of hitting black twice in a row? Of hitting neither red nor black twice in a row? (Give the exact numbers as mathematical expressions, and also calculate approximate values as decimal numbers.)

19. Sam generates a million random bit strings of length 100 (0 and 1 are equally likely in each position). What is the probability that at least one of them has a run of 10 consecutive 0s or 10 consecutive 1s?

20. What are the odds that in a group of 5 people, at least two of them were born on the same day of the week?

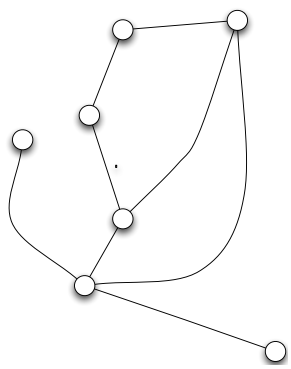
21. Suppose that a drug test has 2% false positives (that is, 2% of the people who are not drug users nonetheless test positive), and no false negatives (no one who tests negative uses drugs). Suppose 1% of the population uses drugs. If you test positive, what are the odds you are actually a drug user?

22. Of the following numbers, which pairs are relatively prime? 21, 24, 28.

23. If you pick a number at random between 1,000,000 and 2,000,000, roughly what are the odds that it is prime?

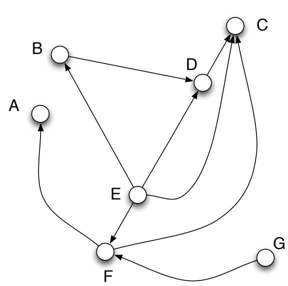
24. (a) What is 199 mod 9?
    (b) What is 199^{100} mod 9?

25. Consider the following undirected graph.



Is it connected? Bipartite? What is the length of the longest cycle, if any? What is the degree of the maximum-degree node? The minimum-degree node? If you remove the articulation point, how many connected components are left?

26. Now consider the following directed graph.



Which nodes are sources? Sinks? Is the graph acyclic? If not, which are the cycles? Write out the edge-list and adjacency matrix representations of the graph. What edges would need to be added to make it the graph of a transitive relation?

27. Assume that no node of a tree can have more than three children. Draw a minimum-height tree with 8 nodes. What is the height of a minimum-height tree with 1000 nodes?

THE END